L15 The Mean-value theorem (均值定理)
4.2 Increasing and decreasing functions (遞增遞減函數)

習題 P158(4.9.12.23.25.26.29.35.39.40.42)

補充題：

Let $f:[a,b]\to\mathbb{R}$ be diff. If $f'(a)>0$ and $f'(b)<0$, then $\exists\ c\in(a,b)$ s.t. $f'(c)=0$.

(Do not assume that f' is cont.)

(Hint:Thm.A)

隨意題：(Intermediate-value property for derivative):

Let $f:[a,b]\to\mathbb{R}$ be diff.

If c is between f'(a) and f'(b), then $\exists\ x_0\in(a,b)$ s.t. $f'(x_0)=c$.

(Do not assume that f' is cont.)

(Hint:Consider F(x)=f(x)-cx, then by 補充題)

Let $F(x)=f(x)-cx$

Then F(x) is cont. on [a,b] and diff. on (a,b).

$\because\ F'(x)=f'(x)-c\ \therefore\ F'(a)=f'(a)-c>0$ and $F'(b)=f'(b)-c<0$

$\because\ F'(a)>0$ and $F'(b)<0\ \therefore$ By 補充題 $\exists\ x_0\in(a,b)$ s.t. $F'(x_0)=0$

$\because\ F'(x)=f'(x)-c=0\ \therefore\ f'(x_0)-c=0\Rightarrow f'(x_0)=c$

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Thm:(Mean-value theorem)

Let $f:[a,b]\to\mathbb{R}$ be a function.

If is cont. on $[a,b]$ and diff. on (a,b),

then $\exists\, c\in(a,b)$ s.t. $f'(c)=[f(b)-f(a)]/(b-a)$ or $f(b)-f(a)=f'(c)(b-a)$.

口語：

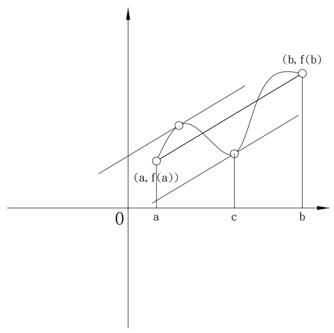
如果有一個函數在閉區間連續且在開區間可微，則存在有一個 c 屬於

ab 開區間使得該點的微分等於頭尾 y 差值除 x 差值(該點切線斜率等

於頭尾割線斜率)或 y 差值等於該點微分乘 x 差值。

pf:使用 Rolle's Thm.

Q:頭尾取値不相等，怎麼修改？A:從圖形上來看，圖形上有兩個函數，$y=f(x)$和頭尾的連線$[y-f(a)]/(x-a)=[f(b)-f(a)]/(b-a)\Rightarrow y=[f(b)-f(a)]/(b-a)\cdot(x-a)+f(a)$，因爲頭尾取値相等，利用兩個函數相減，可以創造頭尾取値相等。



Let $h(x)=f(x)-\{[f(b)-f(a)]/(b-a)\cdot(x-a)+f(a)\}$ 輔助函數，它的性質要自己宣稱。

Then h is cont. on $[a,b]$ and diff. on (a,b). 可以寫更完整，利用微分四則運算。

$\because$ $h(a)=0$ and $h(b)=0$ $\therefore$ By Rolle's thm $\exists\, c\in(a,b)$ s.t. $h'(c)=0$.

$\because$ $h'(x)=f'(x)-[f(b)-f(a)]/(b-a)$ $\therefore$ $h'(c)=f'(c)-[f(b)-f(a)]/(b-a)=0$

$\Rightarrow f'(c)=[f(b)-f(a)]/(b-a)$

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eg1:Show that equation 6x^4-7x+1=0 doesn't have more than two distincr real roots.

pf:

Q:沒有超過三個實根，怎麼證？
A:假設它有三個，得到矛盾。

Let f(x)=6x^4-7x+1

assume that ∃ $x_1<x_2<x_3$ s.t. $f(x_1)=f(x_2)=f(x_2)=0$

Q:誰是主詞？
A:有三個。

∵ f is cont. on $[x_1, x_2]$ and f diff. on (x_1, x_2)

∴ By Rolle's thm., ∃ $c_1\in(x_1, x_2)$ s.t. $f'(c_1)=0$

Similarly, ∃ $c_2\in(x_2, x_3)$ s.t. $f'(c_2)=0$

f'(x)=24x^3-7=0⇒x=(7/24)^(1/3) (→←)

Therefore f doesn't have more than two distincr real roots.

eg2:Prove that the eguation 6x^5+13x+1=0 has exactly one real root.

pf:

Q:洽有？
A:「洽」剛好有。
Q:怎樣證洽有一個實根？
A:先證不會超過一個，再證只有一實根。

Let f(x)=6x^5+13x+1

assume that ∃ $x_1<x_2$ s.t. $f(x_1)=f(x_2)=0$

∵ f is cont. on $[x_1, x_2]$ and f diff. on (x_1, x_2)

∴ By Rolle's thm., ∃ $c \in (x_1, x_2)$ s.t. $f'(c)=0$

But $f'(x)=3x^4+13>0$ (→←)

Therefore f has not more than one real root.

$f(0)=1>0$, $f(-1)=-18<0$

∵ f is cont. on $[-1,0]$

∴ By Intermediate value thm. ∃ $x_0 \in (a,b)$ s.t. $f(x_0)=0$.

⇒f has a root.

Therefore f has exactly one real root.

eg3:Prove that if $f(x)=ax^2+bx+d$, then the number in $[x_1, x_2]$ satisfying the

Mean-value thm. is just the midpoint $c=(x_1+x_2)/2$.

pf:

∵ $f(x)$ is cont. on $[x_1, x_2]$ and diff. on (x_1,x_2)

∴ By Mean-value thm., ∃ $c \in (x_1,x_2)$ s.t. $f'(c)=[f(x_2)-f(x_1)]/(x_2-x_1)$

$=[a(x_2^2-x_1^2)+b(x_2-x_1)]/(x_2-x_1)=a(x_2+x_1)+b$

∵ $f'(x)=2ax+b$ ∴ $f'(c)=2ac+b$

⇒$2ac+b= a(x_2+x_1)+b$

⇒$c=(x_1+x_2)/2$

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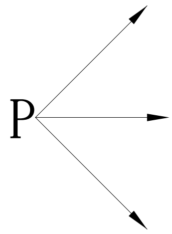
§ 4.2 Increasing and decreasing functions.

學過微積分畫圖不在是個問題
首先，如果一個函數如果不連續，變化是很厲害，不行畫出來。
Q:你會畫怎樣的圖形？A:可以一筆畫完的圖形。



Question:如何劃(連續)函數 $y=f(x)$的圖形？

有三種畫法(往上、水平、往下)，圖形是一段一段畫的，往上或往下。
往上叫遞增函數，往下叫遞減函數。往上往下有高峰，往下往上有低谷。

Answer:若 P 點為函數圖形上的一點

1.遞增、遞減函數的數學建模。

2.高峰、低谷的數學建模。有幾個就畫幾個點

Def:

a function f is said to Increase on the interval I

(表成 f↗on I), if $f(x_1)<f(x_2)$, $\forall\ x_1<x_2$ on I.

a function f is said to decrease on the interval I

(表成 f↘on I), if $f(x_1)>f(x_2)$, $\forall\ x_1<x_2$ on I.

Q:一個函數遞增會不會連續？A:不一定。反過來更扯。

Question:怎樣的函數在 I 上會遞增或遞減？

Thm:Let f be diff. on (a,b).

If $f'>0$ on (a,b), then f↗on (a,b).

如果在 ab 開區間微分大於零，則在 ab 開區間遞增。

If $f'<0$ on (a,b), then f↘on (a,b).

如果在 ab 開區間微分小於零，則在 ab 開區間遞增。 反過來說不對

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pf:

① Let $x_1<x_2$ in (a,b). (To show $f(x_1)<f(x_2)$)

∵ f is diff. on $[x_1,x_2]$ x_1,x_2 屬於 a,b 中的兩個數

∴ f is cont. on $[x_1,x_2]$

By Mean-value thm., $\exists\ c\in(x_1,x_2)$ s.t. $f(x_2)-f(x_1)=f'(c)(x_2-x_1)$

∵ $f'(c)>0$ and $x_2-x_1>0$

∴ $f(x_2)-f(x_1)>0\Rightarrow f(x_2)>f(x_1)$

Therefore f↗on (a,b)